

Circular Sawing Experiments

1576

ON A RADIAL ARM SAW¹

C. W. McMILLIN AND J. L. LUBKIN
Central Research Laboratory,
American Machine and Foundry Co.,
Stamford, Conn.

Net power required to saw certain wood species was determined under various conditions. For flat-sawn hard maple, one formula accurately covered effects of depth of cut, direction of cut, and feed rate.



Fig. 4.—Tooth Configuration of 16-in. combination sawblade used in the experiments.

SEVERAL YEARS AGO, the American Machine and Foundry Co. decided to conduct a comprehensive investigation in the field of cutting processes. The initial emphasis has been on circular sawing because of the product line of the company's DeWalt Division. As a necessary antecedent to theoretical and experimental investigations, an extensive review of the literature on circular sawing was undertaken by Lubkin (1)². Although considerable quantities of information are available on rip sawing, virtually no information was discovered on crosscut or miter sawing of wood. The latter operations are important and are the principal cuts for which radial-arm saws are used.

Accordingly, a broad program of sawing research was established, aimed at objectively defining the basic relationships and principal variables. A program such as this naturally includes consideration of such factors as net cutting power, cutting geometry, surface finish, tool life, and others factors that enter into machinability and equipment design.

The research summarized in this paper is part of this continuing program, which is intended to obtain systematic information on the performance of circular sawblades and circular sawing machines. The effect on net cutting power of such variables as wood species, feed rate, depth of cut, and cutting direction has been measured for a combination-type sawblade. Cutting direction is defined here as the

miter cutting angle in the general sense, which includes all directions from the cross to the rip position. Comparative data are also given on "up" and "down" rip sawing.³ No experimental evaluation of sawed surface quality is included.

Many circular sawing studies presented in the past have failed to report such significant quantities as static and dynamic runout, tooth height variation, and other readily measurable quantities. Such quantities should be systematically measured and reported, so that research results can be evaluated properly by other investigators. The purpose of this research, however, is to obtain data such as might be expected under the comparatively uncontrolled conditions that exist in practice. No effort was made to condition the sawblade to any particular degree of roundness, flatness or sharpness.

Nomenclature

The terminology adopted for use in this study is defined below, in convenient engineering units. Geometrical relations are shown in Fig. 1:

P = net cutting power—hp
 f = feed rate—fpm
 k = actual kerf width—in.



The Authors: Charles McMillin holds a BS in forestry and wood technology from Purdue, and a MS in wood technology from the U. of Michigan. He is a research engineer for AMF. James Lubkin holds BS and MS degrees in mechanical engineering and a PhD in applied mechanics from Columbia Univ. He is a senior research engineer at AMF.

d = workpiece depth (depth of cut)—in.
 R = radius of sawblade—in.
 h = distance from axis of sawblade to workpiece surface—in.
 f = sawblade protrusion beyond the workpiece—in.
 b = arc of sawblade rim engagement in workpiece—in.
 W_s = angle at start of sawblade rim engagement—radians
 W_e = angle at exit of sawblade rim engagement—radians

A, B } constants associated with wood
 C_s, C_e } fiber orientation and structure,
 C_s, C_e } in various cutting power formulas
 G, E }

c = cutting velocity—fpm

p = tooth pitch of sawblade—in.

θ = orientation of sawblade plane with respect to the grain. The reference $\theta = 0^\circ$ is taken as perpendicular to the grain, which is the crosscut direction in straight-grained lumber (Fig. 2). The angle θ is interchangeably referred to as "the cutting direction." $\theta = 0^\circ$ and 90° normally correspond to crosscutting and rip sawing, respectively.

Interchangeable terms that refer to the sense of sawblade rotation relative to the feed are tabulated below:

1	2
(a) up cutting	down cutting
(b) normal sawing	climb sawing
(c) counter sawing	climb sawing
(d) sawing against the feed	sawing with the feed

Experimental Procedure

Design of the Experiments: Five separate experiments were required to establish the desired relations between net cutting power, feed rate, cutting direction, and depth of cut. Except where noted, all sawing was done as

⁴ These definitions are for "climb sawing" and a clockwise rotation in Fig. 1. For "normal" sawing, the direction of rotation is opposite (counterclockwise) and the subscripts of W_s and W_e must be interchanged in the diagram.

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² Numbers in parentheses refer to the Literature Cited at the end of this report.

³ See "Nomenclature" section for other terminology commonly used in practice.

normally performed on radial arm saws. That is, all types of miter cuts, including crosscutting, were "climb" cuts, while rip-sawing was performed as "counter" sawing. The following experimental designs were adopted:

Experiment No. 1

Species—Hard maple (*Acer saccharum* Marsh.), Douglas-fir (*Pseudotsuga Menziesii* (Mirb.) Franco) and sugar pine (*Pinus lambertian* Dougl.)
Feed rate—10, 20, 40 and 60 feet per minute

Cutting direction (θ)—0, 45 and 90 degrees

Experiment No. 2

Species—Hard maple (*Acer saccharum* Marsh.)

Feed rate—10, 20 and 40 feet per minute
Depth of cut— $\frac{1}{8}$, $\frac{1}{4}$, $\frac{1}{2}$, 1, $1\frac{1}{2}$, $1\frac{3}{4}$, $2\frac{1}{8}$, $3\frac{1}{4}$ and $5\frac{1}{8}$ inches

Cutting direction (θ)—0 degrees

Experiment No. 3

Species—Hard maple (*Acer saccharum* Marsh.)

Feed rate—20 feet per minute
Depth of cut— $\frac{1}{4}$, $\frac{1}{2}$, 1, $1\frac{1}{2}$, $1\frac{3}{4}$ inches
Cutting direction (θ)—0, 45 and 90 degrees

Experiment No. 4

Species—Hard maple (*Acer saccharum* Marsh.)

Feed rate—10, 20 and 40 feet per minute
Cutting direction (θ)—90 deg., counter and climb sawing

Experiment No. 5

Species—Hard maple (*Acer saccharum* Marsh.)

Feed rate—10, 20 and 40 feet per minute
Cutting direction (θ)—0, 22.5, 45, 67.5 and 90 (climb sawing) degrees

The following were held constant in all the foregoing experiments:

1. Equilibrium moisture content—sugar pine, 9 percent; Douglas-fir, 9 percent; hard maple, 11 percent.
2. All experimental material was carefully selected from flat-sawn, defect-free, straight-grained lumber.
3. Sawblade protrusion below the workpiece was $\frac{1}{16}$ -in.
4. Rotational speed was 3600 rpm (3450 rpm at 7.5 hp motor load)
5. Sawblade was a 16-inch, flat-ground, combination-type blade.

Three replications, one each on three boards, were performed for each cutting condition listed in the experimental design. This was found to be adequate in a preliminary experiment to yield a standard deviation of 1.73 percent of the mean for the worst case. Thus, by ordinary statistical reckoning a band of ± 3.5 percent of the mean should represent a 95 percent confidence coefficient. This is not to imply that less carefully selected and conditioned stock will not give a larger spread.

Experimental Apparatus: A general view of the experimental area is

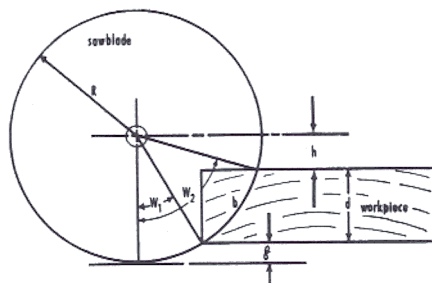


Fig. 1.—Geometrical relations in circular sawing with a radial arm saw. Sawblade is fed from left to right. Clockwise rotation represents climb sawing and counter clockwise rotation corresponds to normal sawing.

shown in Fig. 3. The machine is basically a standard, industrial, radial-arm saw with a 28-inch crossfeed stroke. The sawblade and yoke assembly are driven across the radial arm by an air cylinder equipped with a hydraulic check valve. Preliminary experimentation revealed that no-load feed speeds deviated less than $\frac{1}{2}$ percent from a preset feed rate. The sawblade was driven by a 7.5 hp, 3 phase, 60 cycle, 220/440 volt induction motor. A fixture attached to the saw table facilitated positive clamping of the workpiece. The sawblade was mounted directly on the 1-inch diameter motor arbor by means of the standard 4-inch collar set normally furnished with the machine.

The average power input to the saw motor was measured with a Sanborn AC Wattmeter preamplifier (Model 150-2300). This is one of several standard plug-in units for the Sanborn oscillographic recorder employed in this study.

Sawblade Used: Fig. 4 is a close-up view of the sawblade used in these experiments. The blade is considered to be of excellent commercial quality, and was used in the as-received condition. No effort was made to condition it to any particular degree of roundness, flatness, or sharpness. Typical data for sawblades made by the same manufacturer indicate that the arbor hole will normally be 0.0015 inch above nominal bore size. This type of blade is hand-filed and shows a tooth height range of 0.005 inch on the scoring (high) teeth. It has a 0.010-inch range on the raker (low) teeth, which are about 0.030 inch below the nominal scoring tooth diameter.

During the course of the experiments, the blade was inspected for gummy deposits on the teeth and gullets, which were then removed with carbon tetrachloride. A preliminary experiment with an identical blade revealed that significant dulling effects are not measurable with the comparatively small amount of material sawed in these studies.

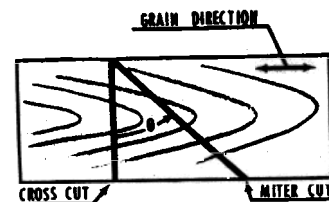


Fig. 2.—Definition of cutting direction (θ). The angle θ gives the orientation of the sawblade plane with respect to the grain. As indicated, $\theta = 0^\circ$ is taken to be perpendicular to the grain (cross-cut direction for straight-grained lumber). Rip-sawing corresponds to $\theta = 90^\circ$.

Actually, two different sawblades of the same type were used in these tests. Experiment 1 was conducted with one blade, and the rest of the studies with a second blade of nominally identical specifications. These are as follows:

diameter—16 in.; type—combination; grind—flat; thickness—0.097 in.; nominal set—0.025 in. per side; actual kerf—0.16 in.; no. of teeth—72; no. of groups—24; no. of scorers—48; collar diameter—4 in.; front & top bevel angle (scorers)— 10° ; clearance angle (scorers)— 30° ; clearance angle (rakers)— 20° ; hook angle (scorers)— 10° ; hook angle (rakers)— 27° ; rakers have no additional bevel angles.

Preparation of Test Material: All experimental material was conditioned for several months in a constant temperature and humidity chamber. This resulted in the equilibrium moisture contents listed in the experimental design. Sufficient test material was then selected from the conditioned stock. Only flat-sawn, defect-free, straight-grained material was used. Two-foot-long test specimens were prepared and suitably randomized, so that each replication of the experiments was made on a different specimen. The experimental material was then surfaced to the required thickness and returned to the conditioning chamber for one week.

Method of Cutting: Before any material was cut, the saw motor was allowed to operate for at least 20 minutes. Experimental material was then removed from the conditioning chamber in small quantities so as to minimize moisture-content changes during cutting. A special fixture was used to assure positive clamping of the workpiece to the saw table. Individual cuts were made 1 inch from the end of the test specimen to avoid the false power readings that may result from cutting too close to a free edge. Moisture content and specific gravity samples were taken during the experiments to detect any substantial variation in these properties.

All rip-sawing operations were performed by the air-hydraulic crossfeed

system, which was employed here for precise and convenient control of the feed rate. In this case, the grain of the lumber is oriented parallel to the radial arm (direction of crossfeed travel). Counter ripsawing is an unusual operation on this type of machine. It is accomplished by rotating the yoke assembly 180° from the normal crosscut position. Cross and miter cuts were performed in the normal manner; that is, by rotation of the radial arm assembly about the vertical column.

Method of Analyzing Power Records: Fig. 5 shows a typical oscillographic power recording. Only the power recorded between points A and B, or the portion of the cutting cycle where the sawblade rim is fully engaged in the workpiece, is considered in establishing the power required to make the cut (see also Fig. 1). A microswitch system is used to indicate these points directly on the oscillographic recording. A straight line is then fitted by eye to determine the average power consumed during this portion of the cut. By applying the watt-meter calibration factor to the average pen deflection thus established, the gross input power consumed is obtained.

To obtain the net cutting power for any given run, it is necessary to subtract system idling losses in a suitable manner. The idling losses on the power records cannot be subtracted directly because motor efficiency changes with load. It is therefore best to use a manufacturer's calibration curve. This was done in the present case. When the data were analyzed, appropriate allowance had thus been made for the individual idling losses of the blade-collar combination used, which amounted to 157 watts here. All the results of this report are given in terms of the net cutting power required for the cutting conditions in question.

Results and Discussion

Power versus Feed Rate: Figs. 6-8 show the power versus feed rate relation obtained for 3 species and 3 cutting directions. Ripsawing here was performed by countersawing. These curves appear normal, and agree in

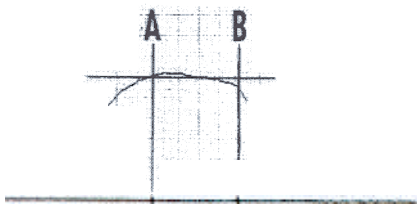


Fig. 5.—Typical motor input power recording. Between points A and B, the sawblade is fully engaged in the workpiece.

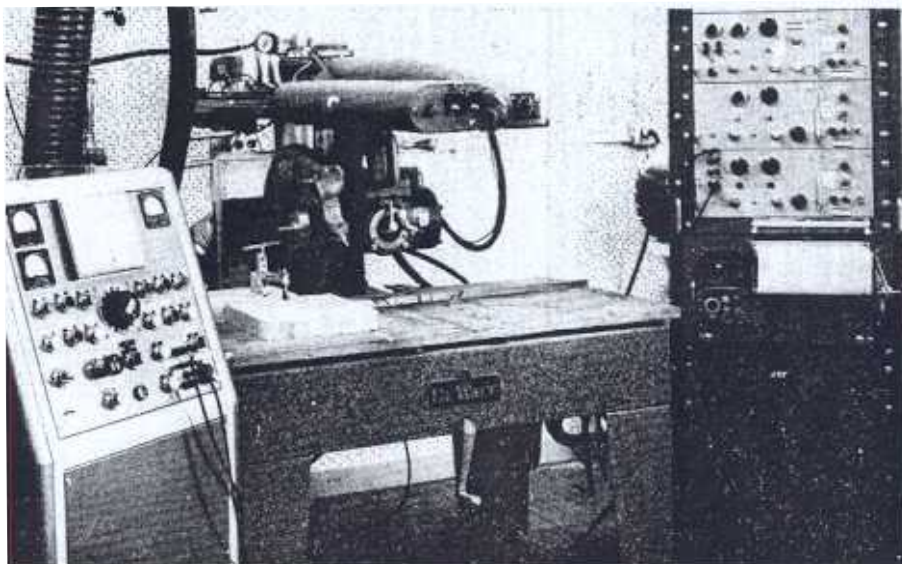


Fig. 3.—General View of Experimental Equipment.

shape with previous sawing studies in both wood and metals, where linear relations are commonly found. See, for example, the wood-sawing studies carried out by Endersby (2) at the British Forest Products Research Laboratory. For the sawing of nonferrous metals, see the research performed by Nuss (4) at DeWalt.

As usual, additional power is required to saw material as the feed rate is increased. When all other factors are held constant, the thickness of the chip that the cutting edge or edges must remove is determined by the feed rate. The chip thickness, in turn, basically determines the power required to detach and remove the chip. There are, however, many additional factors such as wood species, grain orientation, moisture content, and machine and sawblade factors, which may or may not have a pronounced effect.

The relative position of the individual species curves for a given cutting

direction should be noted. In normal ripsawing and 45° miter cutting, the curves are well separated. In crosscutting, sugar pine and Douglas-fir appear to require approximately equal power for the cutting conditions under consideration.

It is assumed that the lumber used is representative of the species under test. Actual specific gravity measurements show that the maple used was 10 percent less dense than the species average, sugar pine was 10 percent high, and the Douglas-fir was very close. This variation, for small samples, is well within the density range to be expected. Within a species, cutting power would be expected to be approximately proportional to the spe-

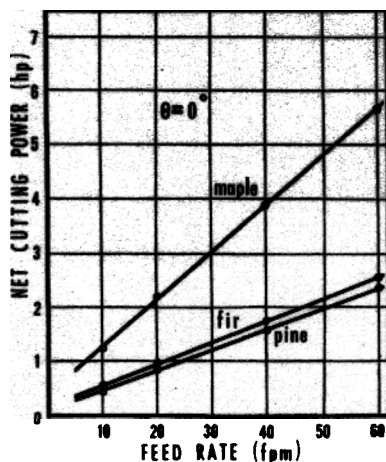


Fig. 6.—Net cutting power versus feed rate when crosscutting three species at 1% in. depth of cut.

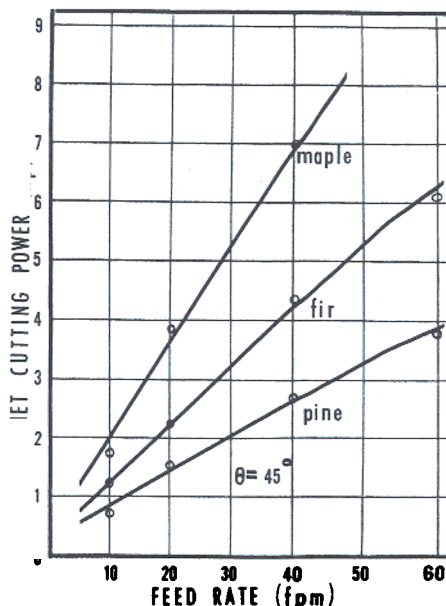


Fig. 7.—Net cutting power versus feed rate when 45° miter sawing three species at 1% in. depth of cut.

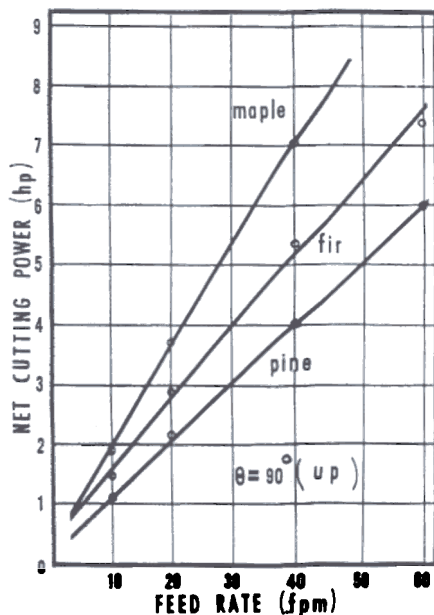


Fig. 8.—Net cutting power versus feed rate when (counter) ripsawing three species at $1\frac{1}{16}$ in. depth of cut.

cific gravity of the particular wood sample involved. If adjustments are made to some arbitrary standard of specific gravity (such as a regional or national average), the relative positions of the curves of Figs. 6–8 will change slightly. For this reason, any cutting-power value presented here or elsewhere cannot be taken as absolute without a close statement of specific gravity, and the many other factors that significantly affect the cutting resistance of wood.

No evaluation of the quality of the sawed surface was made in this study. However, visual examination of specimens indicated that, in general, the greater the feed rate, the poorer the quality of the sawed surface. Since increasing the feed rate increases the feed per tooth, it is reasonable that the sawed surface would reflect increased surface roughness.

Power versus Depth of Cut: Fig. 9 shows the relation of net cutting power to depth of cut for three feed speeds, when hard maple is crosscut. A concave-upward tendency is apparent for small depths of cut, particularly at the slower feed speeds. After a certain small depth of cut is reached, a direct proportionality to workpiece thickness may be considered to exist. These results are in agreement with the experiments of Lotte and Keller (5), who also appear to have worked with a small, fixed protrusion of the sawblade through the workpiece.

The present results are not to be compared directly with the many other studies of effects of workpiece thickness. This is because most other tests have been conducted on types of

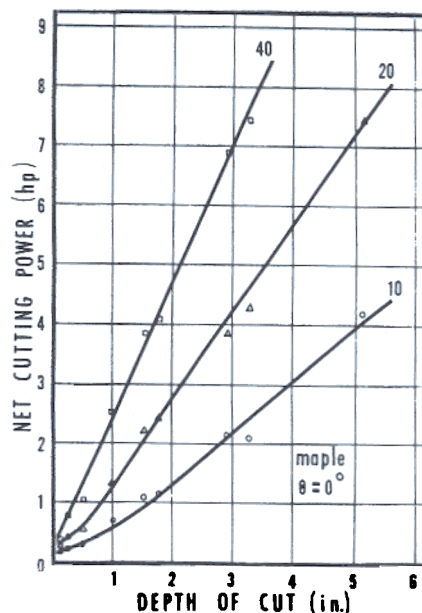


Fig. 9.—Net cutting power versus depth of cut for three feed speeds. Numbers next to curves give feed speed in fpm.

equipment, such as bench or mill saws, where it is normal to keep fixed the distance b between the saw arbor axis and the table or carriage. Thus the experiment is conducted with varying sawblade protrusion as the workpiece thickness is varied, whereas in the present tests and those of Lotte and Keller (5), the sawblade protrusion is held fixed at a small amount. Endersby (2) has given a clear picture of the situation where the table height is held fixed.

It should be pointed out that this entire study has been conducted with carefully selected, flat-sawn lumber. If it is suitably oriented, quarter-sawn lumber may be expected to give slightly different results.

The following formula is given by Harris (6) to determine the power P required for a given set of cutting conditions with a spring-set circular saw of uniform tooth pitch:

$$P = k [Abc/2p + Bfd] \quad (1)$$

Here A and B are constants that depend upon wood properties and orientation relative to the sawblade, f is the feed rate, d is the depth of cut (workpiece thickness), and b is the arc length of sawblade rim engagement. Since the cutting velocity c , the tooth pitch^a p and the kerf width k are constant in this experiment, Eq. (1) may be rewritten with modified constants as:

$$P = C_1b + C_2fd \quad (2)$$

The results shown in Fig. 9 are in agreement with Eq. (2), where P is

^a Although Eq. (1) is strictly applicable to sawblades with uniform tooth pitch, it is clear in principle that it can readily be extended to accommodate combination-type sawblades.

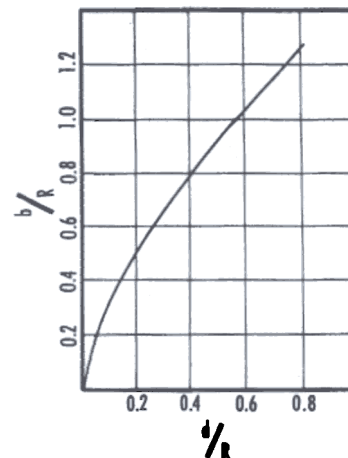


Fig. 10.—Relation of dimensionless arc ratio b/R (angle of saw rim engagement) to dimensionless workpiece depth ratio d/R . Sawblade operated at $1/16$ -in. fixed protrusion ($f/R = 1/128$).

proportional to d in the second term, provided b is also proportional to d in the first term. This would constitute analytical support for the form of the experimental curve.

It can be shown, however, that b is not directly proportional to d for small depths of cut. Radial-arm sawing machines are normally used to saw wood with a small, fixed blade protrusion f . This is generally found to be the most efficient cutting position for wood (1, 5), although elsewhere it is found to be the least efficient for sawing metals (4). The arc of sawblade rim engagement can be calculated on the basis of cutting geometry from the following (see Fig. 1):

$$b = R(W_2 - W_1) \quad (3)$$

Fig. 10 shows the relation of the dimensionless ratios b/R and d/R for the sawblade in question, operated at a fixed protrusion of $1/16$ inch.

Actually, b is not proportional to d . Because of the nature of wood, however, two other factors enter the picture. First of all, the contribution of the first term in Eq. (2) is normally much smaller than that of the second term, at most feed rates of a practical order of magnitude (assuming a reasonably sharp sawblade). On this basis alone it would be reasonable to approximate b by a suitable mean straight line through the origin, for purposes of the present analysis.

Secondly, again because of the nature of wood, parameters such as C_1 and C_2 actually depend upon the grain orientation relative to the cutting edges of the sawblade. In fact, this seems to be the only way to account for previous experimental work, which almost always shows that it is slightly more efficient to ripsaw wood with a small blade protrusion than with a large protrusion. With an argument

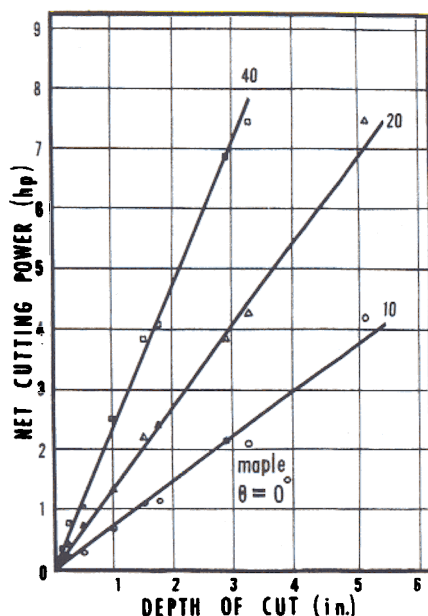


Fig. 11.—Net cutting power versus depth of cut for three feed speeds (test data fitted by straight lines). Numbers next to curves give feed speed in fpm.

along the latter lines, it is possible to give a reasonable explanation for the shapes of the experimental curves in Fig. 9. The minor variations involved, however, are more of academic than of practical significance.

It therefore seems sufficient for the present purposes to do the following. In Eq. (2), consider C_1 and C_2 to be so altered that it can be written approximately:

$$P = d [C_3 + C_4 f] \quad (4)$$

This indicates that net cutting power is a linear function of feed rate, which is adequate to describe the results of most experiments. Also, net power is directly proportional to workpiece depth. This is well supported by the results of Fig. 9, except at very small depths of cut. To show how well the data of Fig. 9 can be represented by straight lines through the origin, the same points are fitted by straight lines in Fig. 11. The fit is seen to be very good.

While Eq. (4) is sufficiently accurate for engineering analysis of the effects of feed rate and depth of cut, it must be remembered that the "constants" C_3 and C_4 actually depend upon wood-fiber orientation relative to the cutting edges. Their values must be obtained by matching the formula in question to suitable experimental results. This is further clarified by the data to be discussed now.

Fig. 12 is a comparison of relation of net cutting power to depth of cut for the rip, 45° miter, and crosscut directions in hard maple. Ripsawing is performed here as up-cutting, while the other operations are performed as

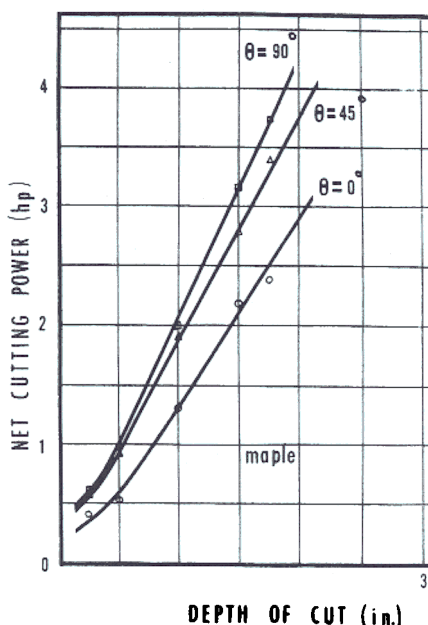


Fig. 12.—Net cutting power versus depth of cut for three cutting directions.

climb sawing (also as usual). These curves have been drawn with a slight concave-upward curvature for small depths of cut, which is the trend shown in Fig. 9. If the reasoning employed in re-drawing Fig. 9 to Fig. 11 is considered, however, it can be seen that a formula such as Eq. (4) will also be sufficiently precise for all cutting directions. Suitable constants C_3 and C_4 must, of course, be determined for the cutting direction in question. In other words, the data of Fig. 12 can likewise be fitted quite accurately by straight lines through the origin.

Effect of Climb versus Counter Ripsawing: Examination of Figs. 6-8 reveals that, under certain cutting conditions, 45° miter sawing appears to require as much (or more) power as ripsawing. The following is an explanation of this apparent inconsistency.

Radial-arm saws normally operate in a climb-type cut for cross and miter sawing for all miter angles short of the rip position. Ripping, however, is usually performed by counter sawing. Climb and counter sawing, as may now be seen, required different amounts of power and hence must be carefully distinguished in all discussions of sawing direction.

Fig. 13 is a comparison of the net cutting power required for ripping hard maple by climb and counter sawing. Climb sawing requires substantially more power than does counter sawing. The amount is about 25 per cent for this case. These results are consistent with studies conducted in Sweden and Norway on bench and table saws (3).

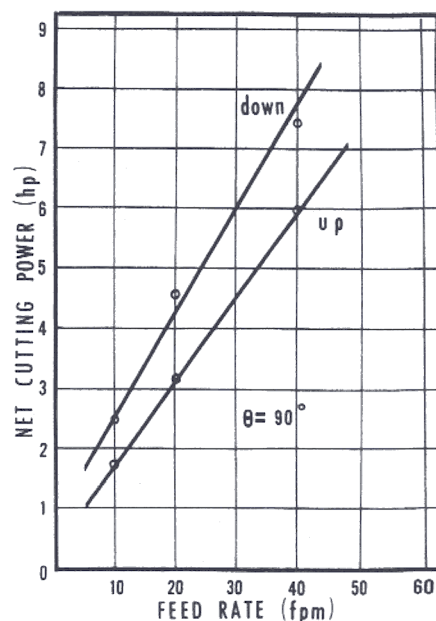


Fig. 13.—Net cutting power versus feed speed for up (normal) and down (climb) ripsawing of 1 1/2 in. hard maple.

The Swedish-Norwegian studies suggest that climb ripsawing is readily practicable if negative hook angles are employed. Cutting powers are then 50 to 70 percent greater than for counter sawing, however, because of the unfavorable (negative) hook. Climb cutting with blades ground with normal positive hook angles is found to be unstable in long cuts because of a tendency to overheat. This is attributed to such factors as 1) a greater mean sawdust movement in the gullet, and 2) less favorable motion of the sawdust while in the gullet of the saw. Both of these factors tend to give more opportunities for the sawdust to escape from the gullet area and increase frictional rubbing. The rubbing action, in turn, generates sufficient heat to produce thermal instability in the blade.

Actual sawdust motion was studied photographically with a special 0.3-microsecond stroboscopic flash camera. The photographs were taken through two transparent plastic plates, which were fixed to the sawblade and rotated with it. Narrow wood specimens were then fed between the plastic plates. Such factors as clearance angle, "shock" loading on tooth entry into the cut, and gullet clogging are also considered as possible causes of overheating, but are discounted on the basis of various theoretical and experimental considerations.

On the basis of the Swedish-Norwegian studies, climb ripsawing with the blade used in the present tests would be expected to show up even less favorably than is indicated by Fig. 13. This is because the present experiments were conducted with very short speci-

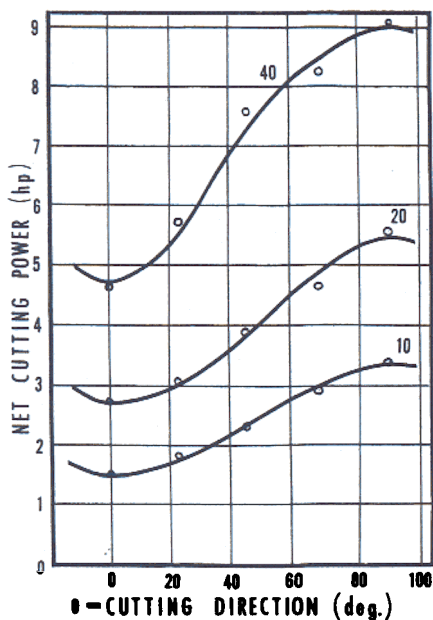


Fig. 14.—Net cutting power versus cutting direction (θ) for three feed speeds. Numbers next to curves give feed speed in fpm.

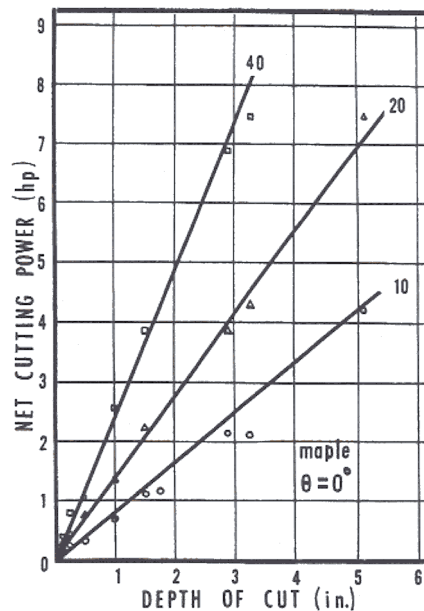


Fig. 15.—Diagram for evaluation of curve-fit: net cutting power versus depth of cut for three feed speeds in fpm. The lines are drawn from Eq. (7). Original test data are also shown.

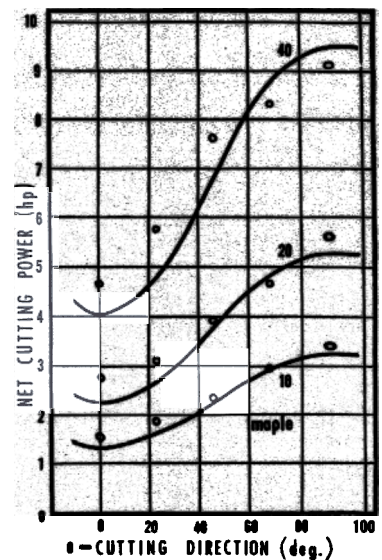


Fig. 16.—Diagram for evaluation of curve-fit: net cutting power versus cutting direction for three feed speeds in fpm. Curves are drawn from Eq. (7). Original test data are also shown.

mens, and heating effects had no chance to become significant.

As the foregoing discussion implies, it is not correct to interpolate between miter and rip sawing in Figs. 7 and 8, since the former is obtained by climb sawing and the latter by counter sawing. Figs. 6 to 8 are perfectly correct, however, for radial-arm saws as normally operated. The question of interpolation between the 45° miter cut and the crosscut positions is treated in much greater detail in the next section.

Cutting Direction: Fig. 14 shows the relation of net cutting power to cutting direction for the blade in question when hard maple is sawn at various feed speeds. In the present case, all cuts (including rip cuts) were performed by climb cutting so that a valid comparison may be made. It follows from Fig. 14 that cutting power in climb sawing can be related to cutting direction by an expression of the form:

$$P = G [1 + E \sin^2 \theta] \quad (5)$$

where G and E are constants and θ is the cutting direction (see Fig. 2 and the Nomenclature Section). For example, $\theta = 0^\circ$ is cross-cutting, θ between 0° and 90° is miter cutting, and $\theta = 90^\circ$ is (climb) rip sawing. Further use will be made of this relation in the next section.

Notice that the curves drawn in Fig. 14, as well as the formula of Eq. (5), imply that cutting power is a symmetrical function of θ at $\theta = 0^\circ$ and $\theta = 90^\circ$. For straight-grained lumber, this must clearly be the case. That is, a miter cut at $\theta = 20^\circ$ must require the same power as a miter cut at $\theta =$

-20° . This implies that the power curve is symmetrical about the $\theta = 0^\circ$ position. The same must also be true of miter cuts near the climb rip position, $\theta = 90^\circ$. That is, a cut at $\theta = 100^\circ$ should require the same power as at $\theta = 80^\circ$.

Derivation of General Power Formula: As previously stated, the power required for a given set of cutting conditions for a spring-set circular saw, can be approximated by Eq. (1). This can be rewritten to Eq. (2) with modified constants when k , c and p are fixed. Although d is not strictly proportional to b for small depths of cut, it has been shown that it is sufficiently accurate to rewrite Eq. (2) as Eq. (4). It has also been shown that power is related to cutting direction in the manner expressed by Eq. (5). It is the purpose of the present section to show that all of these relations can be combined into a single formula with good accuracy.

For a given depth of cut d , feed rate f , and cutting direction θ , it is assumed that net power P is given by the formula

$$P = d [C_s + C_r f] [1 + E \sin^2 \theta] \quad (6)$$

in which C_s , C_r and E are constants to be determined empirically. The constant G of Eq. (5) has been absorbed into C_s and C_r to avoid introducing additional parameters.

By standard methods of curve fitting, it is straightforward to fit actual test data to Eq. (6). The experimental points must be chosen in attempt to match the formula to the actual

behavior of the saw. To show what can be done in this respect, the constants C_s , C_r and E have been evaluated by using all the test results of Figs. 14 and 11 (or its equivalent, Fig. 9), simultaneously. The result is

$$P = d [0.284 + 0.0548 f] \times [1 + 1.343 \sin^2 \theta] \quad (7)$$

On the basis of this formula, Figs. 15 and 16 were drawn, which correspond to Figs. 11 and 14, respectively. The merit of the curve-fit formula can be judged by comparing the positions of the calculated (solid) curves with respect to the original experimental points, which are also shown. When the wide range of feed rates, cutting directions, and depths of cut involved are considered, it can be seen that the formula works rather well.

The curve-fit of Eq. (7) has been obtained by the well-known "method of least squares," a systematic procedure for data analysis in this type of problem. This procedure is indiscriminate in the sense that it gives equal weight to all experimental data. The saw engineer may, in some cases, be willing to sacrifice accuracy at the lower feed rate and depth of cut end of the scale (see Fig. 11), where net cutting power is well below machine capacity and a somewhat larger error can be tolerated. Under these circumstances, it should be possible to improve the fit of the data to the formula elsewhere (see, for example, the left region of Fig. 16). For practical purposes, the present formula is sufficiently precise to be useful. In fitting many variables to a single formula,

one commonly expects some sacrifice in the quality of fit.

To obtain ripsawing powers in counter sawing, values of P calculated from Eq. (7) should be reduced by about 20 percent.

Equation (7) should not be applied to the sawing of quarter-sawn (radial-sawn) lumber, to species other than hard maple, to sawblades other than the 16-inch combination blade tested, and so on. Additional tests will have to be run to determine the constants C_3 , C_4 , and E for these situations. This can be done for a variety of species and sawblades, with a minimum of experimentation, by proper use of statistical techniques.

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